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The maximal profit flow model in designing multiple-production-line system with obtainable resource capacity

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Abstract

A maximal profit flow model for reaching the optimal design of multiple-production-line system undergoing the limitation of obtainable resources is presented in this paper. This model is treated as an extensive maximal flow problem, and an efficient step-by-step algorithm to solve this model is developed. In addition, the concept that the operation cost of a machine does not include only idle or breakdown situations but the maintenance cost also needs consideration is described in this study to extend the applicability. This paper provides a new and practical tool for the production line designer with profound insights.

Keywords: Maximal profit flow model; Multiple-production-line system; Obtainable resources

1. Introduction

To design a good production line is an efficient and profit-enhancement way for manufacturing products. This can be suitable for either manual or automated manufacturing workstations. The increased development of flexible machines offers more options of designing the production line. In practice, a firm has a series of production stages for a given product before manufacturing. A flexible machine [1,2] is capable of combing two or more production stages in one workstation. Hence, a flexible machine can perform a sequence of different operations, but a typical machine [1,2] can only deal with a single operation. Generally, a production line is configured by a sequence of workstations and each workstation has one or more machines of the same type in parallel. This is shown in Fig. 1.

Different workstations may have different production rate for processing. The maximum processing time determines the thorough production rate of the whole line. The workstation that has the maximum

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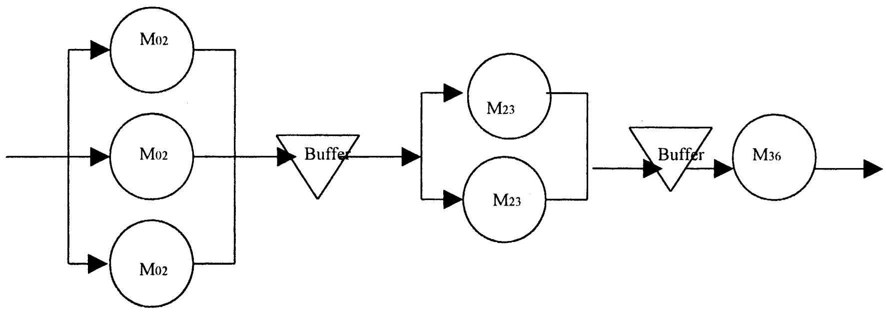

Fig. 1. Schematic diagram showing a flexible production line.

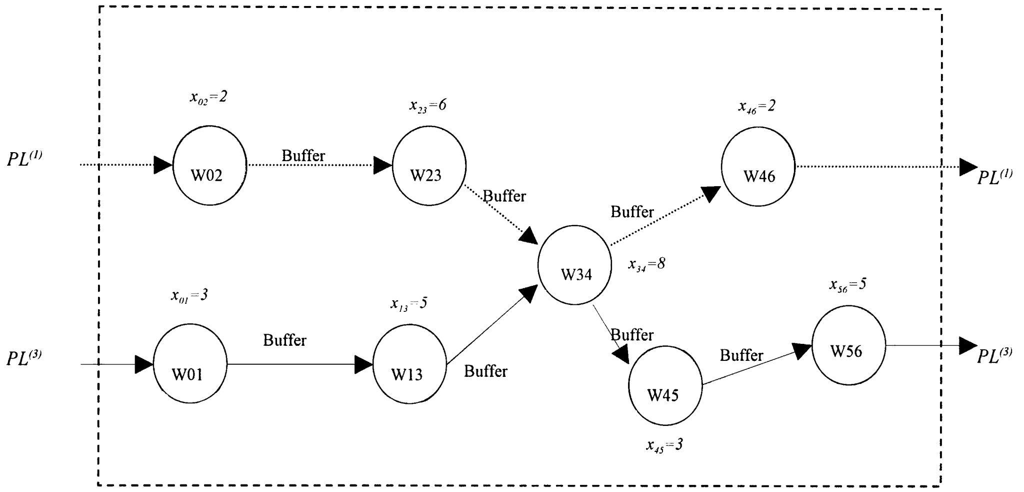

Fig. 2. Schematic diagram showing a multiple-production-line system consists of two automated flexible production lines to manufacture the same product (with six production stages). These two production lines all need W_{34} to manufacture products. Thus, W_{34} from two production lines can be linked together as one workstation.

processing time is called a bottleneck workstation [3]. Thus, the thorough production rate of the line is equal to the output rate of the bottleneck workstation [1–4]. If a station spends less time than the bottleneck workstation in completing the process, it will be idle for the remainder of its production cycle.

In 1998, Adar Kalir and Yohanan Arzi presented their work and proposed the profit-maximizing configuration of workstations (both machine types and number have to be determined) along a production line with typical or flexible machines. The infinite buffers are considered in their study. The main contribution of their study focuses on the unreliable machines and multiple parallel machines (with the same type) which can be introduced in every workstation to reach the maximal profit of the line. The unreliable machine [1,2] means that the machine failure can occur randomly.

Because of different production rates and unreliable machines, workstations may be idle or broken sometime. As per our views, while the machines are in idle or break down state, the operation cost is negligible. This is because the consumption of input resources does not exist, and electricity fees for idle machines are relatively small compared to that of the whole system. In break down situation, the maintenance cost also has to be concerned.

In fact, a production line consisting of flexible machines is very common in today's industry, and there are lots of studies dealing with the optimal design of a production line without the limitation of the machine number. For example, Kalir et al. [1], Johri [3], Tsai and Yao [4], and Martin [5] have presented the optimal way to layout a single-production line. However, the design of multiple-production-line system (shown in Fig. 2) for a given product with the limitation of obtainable resources is rarely discussed. The obtainable resource capacity is provided or evaluated by the firm before designing the system. In this study, the availability of maximum number of machines of each type is regarded as the obtainable resource capacity. To reach the profit-maximizing layout of multiple-production-line system, the reliabilities of machines and obtainable resource capacity are considered simultaneously.

2. Assumption and notation

Before formulating this study, there are several assumptions that should be described as below. They are:

1. The whole multiple-production-line system only makes a same type of product during the manufacturing process and a series of production stages of this product are given. The same workstations from different production lines can be linked together as one workstation in this system.
2. The system consists of automated production lines; every workstation of a line has a specific sequence of production stages and consists of the same type of machines.
3. In every machine, there is only one part that can be processed at one time.
4. No scrapping of parts is considered during the manufacturing process.
5. All products are sold, at their given price, at once after production.
6. The idle or broken machines do not charge for operation.
7. The production lines have enough buffers to avoid blocking and starvation.
8. The cost of buffer space is ignored because it is far less than the operation cost and maintenance cost of the machine; i.e. the buffer space can be large.

The following notation is used in this study:

n: the number of production stages for a given product.

N: $N = \{\text{node } b \mid b = 0, 1, 2, \ldots, n\}$; where node b represents all preceding production stages and stage b have been completed and the production stage $b+1$ is ready to manufacture. In addition, the source node (node 0) only represents that the production stage 1 is ready to process and the sink node (node n) means that all production stages (from 1 to n) have been finished.

MS: the set of all available machine types.

ij: $0 \leqslant i < j \leqslant n$, indicating a workstation functioning from production stage $i+1$ to j in sequence. ij is said to be a feasible workstation if there exists a machine $M_{ij} \in \text{MS}$; where M_{ij} is a machine type of performing a sequence of production stages (from $i+1$ to j).

F: $F = \{ij \mid 0 \leqslant i < j \leqslant n \text{ and } ij \text{ is a feasible workstation}\}$.

t_{ij}: if $ij \in F$, the t_{ij} is the processing time per unit product performed by a single machine M_{ij}; otherwise, t_{ij} is defined to be a given very large number.

l_{ij}: the maximum available number of machine type M_{ij} in the firm; if ij is not feasible, l_{ij} is defined to be 0.

r_{ij}: the reliability of machine M_{ij} which is defined by $r_{ij} = \varepsilon_{ij}/(\varepsilon_{ij} + \delta_{ij})$; where ε_{ij} and δ_{ij} are the mean time between failures and mean time to fix a single machine type M_{ij} respectively.

c_{ij}: operation cost (dollar(s) per unit time) of a single machine type M_{ij}.

c^{f}_{ij}: maintenance cost (dollar(s) per unit time) of a single machine type M_{ij}. That is $c^{f}_{ij}\delta_{ij}$ is the mean maintenance fees of a single machine type M_{ij}.

p: contribution per unit product (the operation and maintenance costs related to the multiple-production-line system are not excluded).

PL: $\text{PL} = \{i_0 i_1, i_1 i_2, i_2 i_3, \ldots, i_{r-1} i_r\}$ is a production line which indicates a sequence of feasible workstations, $i_0 i_1, i_1 i_2, i_2 i_3, \ldots, i_{r-1} i_r$, where $0 = i_0 < i_1 < i_2 < i_3 \cdots < i_r = n$.

$\pi(\text{PL})$: unit profit of a production line which is defined as

$$\pi(\text{PL}) = p - \sum_{ij \in \text{PL}} \left\{ c_{ij} t_{ij} + c^{\text{f}}_{ij} \left(\frac{t_{ij}}{r_{ij}} - t_{ij} \right) \right\} = p - \sum_{ij \in \text{PL}} t_{ij} \left[c_{ij} + c^{\text{f}}_{ij} \left(\frac{1}{r_{ij}} - 1 \right) \right].$$

$\text{PL}^{(l)}$: the lth shortest path of Network (N, F, D) [6,7], where

$$D = \left\{ d_{ij} \mid ij \in F \text{ and } d_{ij} = t_{ij} \left[c_{ij} + c^{\text{f}}_{ij} \left(\frac{1}{r_{ij}} - 1 \right) \right] \right\}$$

is the set of distances among Network (N, F, D) and $\pi(\text{PL}^{(l)})$ can be shown as follows:

$$\pi(\text{PL}^{(1)}) \geqslant \text{II}(\text{PL}^{(2)}) \geqslant \cdots \geqslant \pi(\text{PL}^{(m)}), \tag{1}$$

where m is the number of all feasible production lines.

$\bar{m}$: $\bar{m} = \max\{l \mid 1 \leqslant l \leqslant m, \text{ and } \pi(\text{PL}^{(l)}) > 0\}$. (2)

U_l: the thorough production rate of production line $\text{PL}^{(l)}$.

Decision variables

z^l_{ij}: $z^l_{ij} = 1$ if $ij \in \text{PL}^{(l)}$; (3)

otherwise $z^l_{ij} = 0$. In addition, $z^l_{ij} U_l$ presents the operational production rate of workstation ij in $\text{PL}^{(l)}$.

PR_{ij}: the operational production rate of workstation ij in the multiple-production-line system. Note that, given $U_1, U_2, \ldots, U_m \geqslant 0$, if PR_{ij} is defined by

$$\text{PR}_{ij} = \sum_l z^l_{ij} U_l, \quad \text{for } ij \in F. \tag{4}$$

Then,

$$\sum_i \text{PR}_{ik} = \sum_j \text{PR}_{kj} \quad \forall k = 1, 2, \ldots, n-1, \text{ and } \sum_j \text{PR}_{0j} = \sum_i \text{PR}_{in}. \tag{5}$$

x_{ij}: number of parallel machine type M_{ij} of workstation ij in the multiple-production-line system.

Z: profit of the multiple-production-line system.

3. Model development

Whole feasible configurations of workstations for producing a given product can form a network model, and each feasible configuration can be treated as one feasible path from source-to-sink node [6,7]. Thus, using (5), the problem of designing multiple-production-line system for a given product presented by this study can be transformed to become an extensive maximal flow problem called maximal profit flow (MPF) model. It is listed below.

Objective function (6) presents the profit of the multiple-production-line system. In addition, $t_{ij} c_{ij}$ and $c^{\text{f}}_{ij} t_{ij} (1/r_{ij} - 1)$ are the mean operation cost and mean maintenance cost per unit of workstation ij, respectively. Constraint (7) means that total input operational production rate in node k is equal to its total output rate.

Constraint (8) shows that $\sum_i \mathrm{PR}_{in}$ and $\sum_j \mathrm{PR}_{0j}$ are both equal to the total thorough production rate of the whole system. Final constraint (9) presents that the operational production rate of workstation ij is not greater than its upper bound $l_{ij}r_{ij}/t_{ij}$. In fact, while $\pi(\mathrm{PL}^{(l)})$is a fixed constant for every l, Eq. (6) is changed as $\pi(\mathrm{PL}^{(l)})\mathrm{Max}\sum_j \mathrm{PR}_{0j}$ (the proof is shown in Appendix A) and becomes a traditional maximal flow problem.

$$\text{MPF model:}\begin{cases} \underset{\mathrm{PR}_{ij}}{\mathrm{Max}} \left\{\left(p\sum_j \mathrm{PR}_{0j}\right) - \sum_{ij} t_{ij}\left[c_{ij} + c_{ij}^{\mathrm{f}}\left(\dfrac{1}{r_{ij}} - 1\right)\right]\mathrm{PR}_{ij}\right\}, & (6)\\ \text{s.t.} & \\ \sum_i \mathrm{PR}_{ik} = \sum_j \mathrm{PR}_{kj}, \quad \forall k = 1, 2, \ldots, n-1, & (7)\\ \sum_i \mathrm{PR}_{in} = \sum_j \mathrm{PR}_{0j}, & (8)\\ 0 \leqslant \mathrm{PR}_{ij} \leqslant \dfrac{l_{ij}r_{ij}}{t_{ij}}. & (9) \end{cases}$$

4. Step-by-step algorithm

After introducing the objective function and its associated constraints, the algorithm for reaching optimal solution is proposed. Note that, the set PL forms a path between nodes 0 and n. Hence, the following is the step-by-step algorithm.

Step 0: Let $C_1 \equiv \{C_{ij}^{(1)} | ij \in F$ and $C_{ij}^{(1)} = l_{ij}r_{ij}/t_{ij}\}$, be the set of arc capacities at initial state (denoted by state 1) and U_1^*, $U_1^* = \min_{ij \in \mathrm{PL}^{(1)}} C_{ij}^{(1)} \geqslant 0$, be the maximum flow throughout $\mathrm{PL}^{(1)}$ in Network (N, F, C_1). Save U_1^* and initialize $\alpha = 2$; go to step 1.

Step 1: $C_\alpha \equiv \{C_{ij}^{(\alpha)} | C_{ij}^{(\alpha)} = C_{ij}^{(\alpha-1)} - z_{ij}^{\alpha-1}U_{\alpha-1}^*$, for $ij \in F\}$;

U_α^*, $U\alpha^* = \min_{ij \in \mathrm{PL}^{(\alpha)}} C_{ij}^{(\alpha)} \geqslant 0$, be the maximum flow throughout $\mathrm{PL}^{(\alpha)}$ in Network (N, F, C_α). Then, save U_α^*.

Step 2: If $\alpha = \bar{m}$, stop and go to step 3.

Otherwise, set $\alpha = \alpha + 1$; return to step 1.

Step 3: The optimal solution of the MPF model is obtained as follows:

$$\mathrm{PR}_{ij}^* = \sum_{l=1}^{\bar{m}} z_{ij}^l U_l^*, \quad \forall ij \in F,$$

$$x_{ij}^* = \left[\frac{\mathrm{PR}_{ij}^* t_{ij}}{r_{ij}}\right]^+, \quad \forall ij \in F, \qquad Z^* = \sum_{l=1}^{\bar{m}} U_l^* \pi(\mathrm{PL}^{(l)}).$$

In the following section, let

$$U_l^* = 0 \quad \forall l = \bar{m}+1, \bar{m}+2, \ldots, m. \tag{10}$$

5. The proof of optimization

Let $\{\mathrm{PR}^*_{ij} \mid ij \in F\}$ be the solution set obtained from the algorithm proposed in above section and $\{\mathrm{PR}_{ij} \mid ij \in F\}$ be a given feasible solution set of MPF model. According to (3) and (4), it is valid thatPR^*_{ij} can be presented by U^*_l, and U_l can represent PR_{ij}. It is shown in Table 1.

From the algorithm suggested in this paper, it can be shown that

$$U^*_1 \geqslant U_1, U^*_1 + U^*_2 \geqslant U_1 + U_2, \ldots, \quad \text{and} \quad U^*_1 + U^*_2 + \cdots + U^*_{\bar{m}} \geqslant U_1 + U_2 + \cdots + U_{\bar{m}}. \tag{11}$$

Hence,

$$\begin{aligned} Z(\{\mathrm{PR}^*_{ij}|ij \in F\}) - Z(\{\mathrm{PR}_{ij}|ij \in F\}) &= \sum_l U^*_l \pi(\mathrm{PL}^{(l)}) - \sum_l U_l \pi(\mathrm{PL}^{(l)}) \\ &= \sum_l (U^*_l - U_l)\pi(\mathrm{PL}^{(l)}) \\ &= \sum_{l \leqslant \bar{m}} (U^*_l - U_l)\pi(\mathrm{PL}^{(l)}) + \sum_{l > \bar{m}} (U^*_l - U_l)\pi(\mathrm{PL}^{(l)}); \text{ using (1) and (10)} \\ &\geqslant \sum_{l \leqslant \bar{m}} (U^*_l - U_l)\pi(\mathrm{PL}^{(\bar{m})}) + \sum_{l > \bar{m}} (-U_l)\pi(\mathrm{PL}^{(l)}); \text{ using (2) and (11)} \\ &\geqslant 0. \end{aligned}$$

This proves that the solution obtained from the proposed algorithm in this paper is optimal. In addition, a simple numerical example is illustrated and described in Appendix B.

6. Conclusions

Earlier studies almost focued on discussing the single-line design, unlike the designing of multiple-production-line system with obtainable resources a new trial in this study. Designing a multiple-production-line system has to consider operation cost, maintenance cost, and the reliability of each machine type. Also, the obtainable machine number, type and the production rate of bottleneck workstation in each feasible production line are taken into consideration simultaneously. This design is truly a complicated and hard-solving issue. But, through MPF model, the above issue becomes concrete and easy-to-solve.

In this paper, the special application of lth shortest-route problem is introduced as a useful tool to find out the feasible configurations of workstations. Definitely, the MPF model extends the applicability of maximal flow problem. In addition, whenever $\bar{m} = m$, the optimal solution of MPF model in this study should be among the optimal solution(s) for the traditional maximal flow problem. Moreover, after achieving the

Table 1
Solutions from proposed algorithm and given

Unit profit of $\mathrm{PL}^{(l)}$	Solution from proposed algorithm	Given a feasible solution
$\pi(\mathrm{PL}^{(1)})$	U^*_1	U_1
$\pi(\mathrm{PL}^{(2)})$	U^*_2	U_2
$\vdots$	$\vdots$	$\vdots$
$\pi(\mathrm{PL}^{(l)})$	U^*_l	U_l
$\vdots$	$\vdots$	$\vdots$
$\pi(\mathrm{PL}^{(m)})$	U^*_m	U_m

optimal solution of MPF model, the total operation cost and total maintenance cost of machines in the whole system can be computed. The proportion of them functions as an index in cost analysis. Furthermore, adding extra machines into the workstation(s), located in $\mathrm{PL}^{(l)}$ with smaller l, where its (their) associated obtainable resources have been totally consumed, is considered as a feasible way to enlarge the system profit. In sum, this study provides a practical way to a firm to evaluate the optimal thorough production rate, profit, and the design of multiple-production-line system with available machine number and types before manufacturing.

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Appendix A

While $\pi(\mathrm{PL}^{(l)})$ is a fixed constant for every l, Eq. (6) is changed as

$$\pi(\mathrm{PL}^{(l)})\mathrm{Max}\sum_{j}\mathrm{PR}_{0j}$$

Let $T_{ij} = t_{ij}(c_{ij} + c^{\mathrm{f}}_{ij}(1/r_{ij} - 1))$, Eq. (6) can be rewritten as

$$\begin{aligned}\underset{\mathrm{PR}_{ij}}{\mathrm{Max}}\left\{p\sum_{j}\mathrm{PR}_{0j} - \sum_{ij}T_{ij}\mathrm{PR}_{ij}\right\} &= \underset{U_l}{\mathrm{Max}}\left\{p\sum_{l}z^{l}_{0j}U_l - \sum_{ij}T_{ij}\sum_{l}z^{l}_{ij}U_l\right\}\\ &= \underset{U_l}{\mathrm{Max}}\sum_{l}U_l\left\{p - \sum_{ij}T_{ij}z^{l}_{ij}\right\}\\ &= \underset{U_l}{\mathrm{Max}}\sum_{l}U_l\pi(\mathrm{PL}^{(l)});\ \text{since } \pi(\mathrm{PL}^{(l)}) \text{ is a fixed constant for } l = 1, 2, \ldots, m\\ &= \pi(\mathrm{PL}^{(l)})\underset{U_l}{\mathrm{Max}}\sum_{l}U_l\\ &= \pi(\mathrm{PL}^{(l)})\mathrm{Max}\sum_{j}\mathrm{PR}_{0j}.\end{aligned}$$

Appendix B. The numerical example

A product consisting of seven production stages is considered in this example. The obtainable resource capacity l_{ij} and associated information of each machine type are described in Table 2.

$p = \$65$, then apply lth shortest-route method to list paths according to the order.

$$\begin{aligned}&\pi(\mathrm{PL}^{(1)}) = 10.217(\mathrm{PL}^{(1)} = \{01, 13, 36, 67\})\\ &\geqslant \pi(\mathrm{PL}^{(2)}) = 5.727\ (\mathrm{PL}^{(2)} = \{01, 12, 25, 56, 67\})\\ &\geqslant \pi(\mathrm{PL}^{(3)}) = 4.403(\mathrm{PL}^{(3)} = \{01, 12, 23, 36, 67\})\\ &\geqslant \pi(\mathrm{PL}^{(4)}) = 2.438(\mathrm{PL}^{(4)} = \{01, 13, 34, 45, 56, 67\})\\ &\geqslant \pi(\mathrm{PL}^{(5)}) = 1.997(\mathrm{PL}^{(5)} = \{01, 13, 34, 46, 67\}) > 0.\end{aligned}$$

Table 2
The data of numerical example

Indices ij	Machine types M_{ij}	Processing time t_{ij} (h)	Reliability r_{ij}	Maximum available number of machines l_{ij}	Operation cost c_{ij} ($/h)	Maintenance cost c^{f}_{ij} ($/h)
01	M_{01}	0.44	0.90	8	20.0	8.5
12	M_{12}	1.10	0.85	8	14.3	8.0
23	M_{23}	0.30	0.90	8	15.0	8.0
34	M_{34}	0.90	0.90	8	21.7	8.6
45	M_{45}	0.50	0.90	8	20.0	8.5
56	M_{56}	0.18	0.95	8	18.3	8.0
67	M_{67}	0.15	0.90	8	18.3	8.0
13	M_{13}	0.52	0.90	6	30.0	11.0
25	M_{25}	1.00	0.95	6	26.0	10.0
36	M_{36}	1.20	0.90	6	21.0	9.4
46	M_{46}	0.60	0.95	6	23.3	9.6

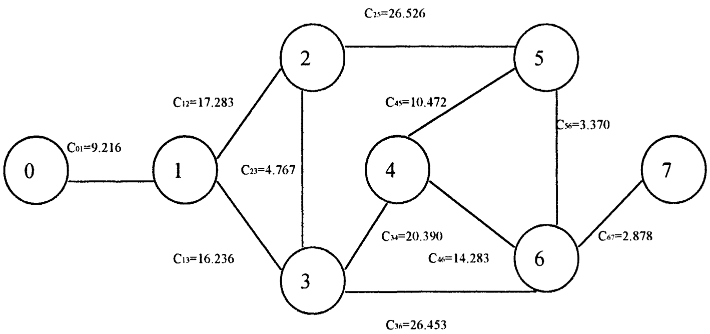

Fig. 3. Network (N, F, C_1).

(I) Compute the thorough production rate of $\mathrm{PL}^{(1)}$:
Step 0: $\bar{m} = 5$.
Draw a Network (N, F, C_1) (shown in Fig. 3)

$$C_1 = \left\{ \begin{matrix} C^{(1)}_{01} = 16.36, & C^{(1)}_{12} = 6.18, & C^{(1)}_{13} = 10.38, & C^{(1)}_{23} = 24.00, \\ C^{(1)}_{25} = 5.70, & C^{(1)}_{34} = 8.00, & C^{(1)}_{36} = 4.50, & C^{(1)}_{45} = 14.40, \\ C^{(1)}_{46} = 9.50, & C^{(1)}_{56} = 42.22, & C^{(1)}_{67} = 48.00 & \end{matrix} \right\}$$

and $U^*_1 = 4.50$.
(II) Compute the thorough production rate of $\mathrm{PL}^{(2)}$
Step 1:

$$C_2 = \left\{ \begin{matrix} C^{(2)}_{01} = 11.86, & C^{(2)}_{12} = 6.18, & C^{(2)}_{13} = 5.88, & C^{(2)}_{23} = 24.00, \\ C^{(2)}_{25} = 5.70, & C^{(2)}_{34} = 8.00, & C^{(2)}_{36} = 0, & C^{(2)}_{45} = 14.40, \\ C^{(2)}_{46} = 9.50, & C^{(2)}_{56} = 42.22, & C^{(2)}_{67} = 43.50 & \end{matrix} \right\}$$

and $U_2^* = 5.70$.
Step 2: $\alpha = 2 + 1 = 3$, return to Step 1.
(III) Compute the thorough production rate of $PL^{(3)}$
Step 1:

$$C_3 = \begin{Bmatrix} C_{01}^{(3)} = 6.16, & C_{12}^{(3)} = 0.48, & C_{13}^{(3)} = 5.88, & C_{23}^{(3)} = 24.00, \\ C_{25}^{(3)} = 0, & C_{34}^{(3)} = 8.00, & C_{36}^{(3)} = 0, & C_{45}^{(3)} = 14.40, \\ C_{46}^{(3)} = 9.50, & C_{56}^{(3)} = 36.52, & C_{67}^{(3)} = 37.80 & \end{Bmatrix}$$

and $U_3^* = 0$.
Step 2: set $\alpha = 3 + 1 = 4$, then return to Step 1.
(IV) Compute the thorough production rate of $PL^{(4)}$
Step 1: $C_4 = C_3$
and $U_4^* = 5.88$.
Step 2: set $\alpha = 4 + 1 = 5$, then return to Step 1.
(V) Compute the thorough production rate of $PL^{(5)}$
Step 1:

$$C_5 = \begin{Bmatrix} C_{01}^{(5)} = 0.28, & C_{12}^{(5)} = 0.48, & C_{13}^{(5)} = 0, & C_{23}^{(5)} = 24.00, \\ C_{25}^{(5)} = 0, & C_{34}^{(5)} = 2.12, & C_{36}^{(5)} = 0, & C_{45}^{(5)} = 8.52, \\ C_{46}^{(5)} = 9.50, & C_{56}^{(5)} = 30.64, & C_{67}^{(5)} = 31.92 & \end{Bmatrix}$$

and $U_5^* = 0$.

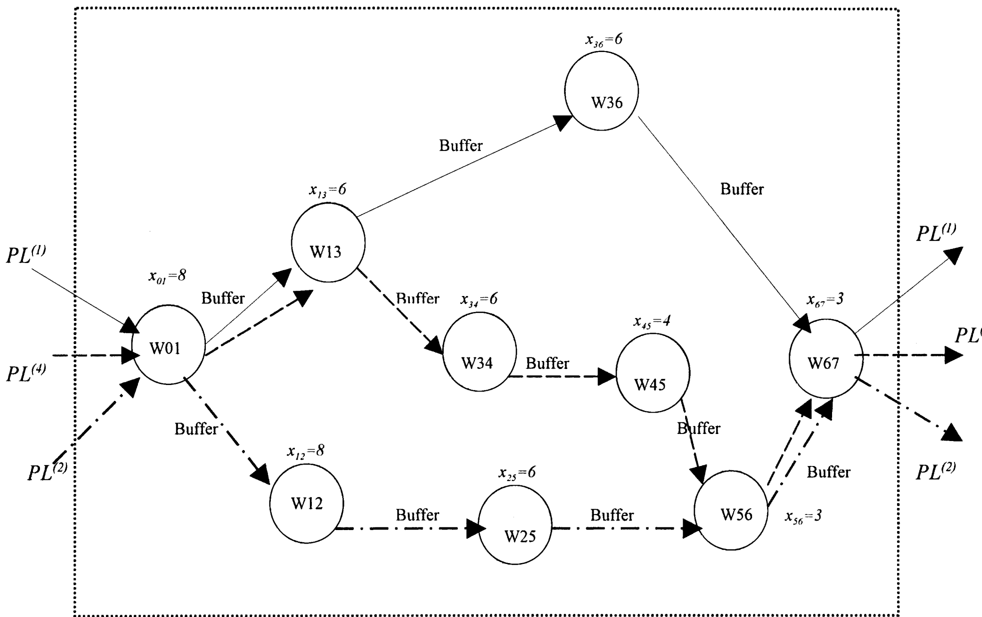

Fig. 4. Schematic diagram showing the optimal design of the numerical example in Appendix B. There are three production lines in this system to manufacture the same products.

Step 2: $\alpha = \bar{m} = 5$, go to Step 3.
Step 3: $PR^*_{01} = 16.08$; $PR^*_{12} = 5.70$; $PR^*_{13} = 10.38$; $PR^*_{23} = 0$; $PR^*_{25} = 5.70$; $PR^*_{34} = 5.88$; $PR^*_{36} = 4.50$; $PR^*_{45} = 5.88$; $PR^*_{46} = 0$; $PR^*_{56} = 11.58$; $PR^*_{67} = 16.08$ $x^*_{01} = 8$, $x^*_{12} = 8$, $x^*_{13} = 6$, $x^*_{23} = 0$, $x^*_{25} = 6$, $x^*_{34} = 6$, $x^*_{36} = 6$, $x^*_{45} = 4$, $x^*_{46} = 0$, $x^*_{56} = 3$, $x^*_{67} = 3$.

$$Z^* = 92.9558, \sum_{j=1}^{7} PR_{oj} = 16.08.$$

According to the illustration above, there are three flexible production lines in the optimal design of multiple-production-line system. They are $PL^{(1)}$, $PL^{(2)}$, and $PL^{(4)}$. These three lines all begin from workstation 01. Then, $PL^{(1)}$ and $PL^{(4)}$ go to workstation 13 together and $PL^{(2)}$ goes to 12. After workstation 13, $PL^{(1)}$ goes to station 36, and $PL^{(4)}$ gets into 56 through stations 34 and 45. Regarding $PL^{(2)}$, after passing through workstation 12, it goes to station 25 and then gets into 56. Finally, all the three production lines get into workstation 67 to complete the multiple-production-line system. The thorough production rate of this system is 16.08 unit per hour. Under such a design, the profit obtained from this system is 92.9558 $/h. The layout of this system and the machine number of each workstation are shown in Fig. 4.

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